Axion-Photon Couplings

Quantization and non-standard axion electrodynamics

Matt Reece, October 3, 2023 “COSMIC WISPers” talk

Based on: 2309.03939, and 2309.07951 with Ben Heidenreich & Jake McNamara

[Also see 2309.03934 Agrawal, Platschorre; 2309.03937 Choi, Forslund, Lam, Shao; 2309.05636 Cordova, Hong, Wang]

Axion electrodynamics

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - g_{a\gamma\gamma} \boldsymbol{B} \cdot \boldsymbol{\nabla} a \,, \quad \boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} - \boldsymbol{J}_{\mathrm{M}} \,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = \rho_{\mathrm{M}} \,, \quad \boldsymbol{\nabla} \times \boldsymbol{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{J} - g_{a\gamma\gamma} \left(-\boldsymbol{B} \frac{\partial a}{\partial t} + \boldsymbol{E} \times \boldsymbol{\nabla} a \right) .$$

These are the modified Maxwell equations as derived by Sikivie, with added terms for magnetic monopole charges and currents.

They are the starting point for nearly all axion experiments.

Is this the whole story, or not?

Standard axion electrodynamics

Gauge transformations: discrete shifts of axion, U(1) gauge transformations

$$\theta \cong \theta + 2\pi \qquad\qquad A_\mu \mapsto A_\mu - \mathrm{i}\mathrm{e}^{-\mathrm{i}\alpha(x)}\partial_\mu \mathrm{e}^{\mathrm{i}\alpha(x)} = A_\mu + \partial_\mu \alpha(x).$$

Action:

$$\int \mathrm{d}^4x \, \sqrt{|g|} \left(-\frac{1}{2} f^2 \partial_\mu \theta \partial^\mu \theta - \frac{1}{4e^2} F_{\mu\nu} F^{\mu\nu} \right) + \frac{n}{8\pi^2} \int \theta F \wedge F,$$

This implies a quantization condition:

$$n \in \mathbb{Z}$$

Let’s give a very explicit derivation.

Gauge-dependent action and quantized coupling

Consider a Euclidean action: $S_E = \mathrm{i}\, c \int \theta\, F \wedge F.$

Under $\theta \mapsto \theta + 2\pi,\quad S_E \mapsto S'_E = S_E + 2\pi\mathrm{i}\, c \int F \wedge F.$ (Chern-Simons term)

But all we need to define a QFT is that $\exp\left[-S_E\right]$ be well-defined.

Luckily there is a quantization condition: $\frac{1}{8\pi^2} \int_M F \wedge F \in \mathbb{Z},$ *M* closed, spin.

$$\exp\left[-S_E\right] = \exp\left[-S'_E\right] \text{ implies } c = \frac{k}{8\pi^2}, \quad k \in \mathbb{Z}.$$

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Re-examining assumption #1

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Field configuration with flux in QED

Easy to write down very explicit examples. Flux on a 2-torus:

$A = Cx_1 \mathrm{d}x_2$ inconsistent with periodicity $x_1 \cong x_1 + 2\pi r_1$

unless $A' - A = 2\pi C r_1 \mathrm{d}x_2 = -\mathrm{i}e^{-\mathrm{i}\alpha(x)}\mathrm{d}e^{\mathrm{i}\alpha(x)}$ for some well-defined U(1) transformation. x_2 itself is not well-defined, but the winding transformation $e^{\mathrm{i}nx_2/r_2} \in \mathrm{U}(1)$ *is*. Hence $C = n/(2\pi r_1 r_2)$ are the allowed values, with

$$\frac{1}{2\pi}\int F = n \in \mathbb{Z}\,.$$

Field configuration with $\int F \wedge F \neq 0$ in QED

Now, on a 4-torus, just take two terms of this type:

$$A = \frac{n_1}{2\pi r_1 r_2} x_1 \mathrm{d}x_2 + \frac{n_2}{2\pi r_1 r_2} x_3 \mathrm{d}x_4, \qquad n_{1,2} \in \mathbb{Z}$$

This is a valid gauge field configuration and it has $\frac{1}{8\pi^2}\int F \wedge F = n_1 n_2 \,.$

Field configuration with $\int F \wedge F \neq 0$ in Standard Model

The minimal Standard Model gauge group $[\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)]/\mathbb{Z}_6$ correlates charges. Below the EWK scale, $\mathrm{U}(3)$ with $3Q \equiv -n_3 \mod 3$.

$$C = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\left(\frac{1}{2\pi r_1 r_2} x_1 \mathrm{d}x_2 + \frac{1}{2\pi r_3 r_4} x_3 \mathrm{d}x_4\right), \quad A = \frac{1}{2\pi r_1 r_2} x_1 \mathrm{d}x_2 + \frac{1}{2\pi r_3 r_4} x_3 \mathrm{d}x_4$$

is a valid gauge field on the 4-torus with topological charges

$$\frac{1}{8\pi^2}\int \mathrm{tr}(G \wedge G) = \frac{2}{3}, \quad \frac{1}{8\pi^2}\int F \wedge F = 1.$$

From the viewpoint of SU(3) or U(1) individually these are fractional.

[see, e.g., ’t Hooft 1979; Anber, Poppitz 2110.02981]

Quantization of axion couplings in the SM

Given the axion couplings

$$I_{\rm ax} = \int \left[\frac{k_G}{8\pi^2}\,\theta\,{\rm tr}(G\wedge G) + \frac{k_F}{8\pi^2}\,\theta\,F\wedge F\right]$$

the existence of a field configuration with

$$\frac{1}{8\pi^2}\int {\rm tr}(G\wedge G) = \frac{2}{3}, \quad \frac{1}{8\pi^2}\int F\wedge F = 1.$$

implies

$$\frac{2}{3}k_G + k_F \in \mathbb{Z}.$$

Interestingly, $E/N \equiv 2k_F/k_G = 8/3$ gives rise to the minimal $|g_{a\gamma\gamma}|$ if $k_G = 1$.

Re-examining assumption #2

Hidden assumption #2: F is invariant under $\theta \mapsto \theta + 2\pi$

SL(2,Z) electric-magnetic duality

Free U(1) gauge theory has an $\mathrm{SL}(2,\mathbb{Z})$ electromagnetic duality.

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} {A_{\mathrm{M}}}' \\ A' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A_{\mathrm{M}} \\ A \end{pmatrix} \qquad \begin{pmatrix} {J_{\mathrm{E}}}' \\ -{J_{\mathrm{M}}}' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} J_{\mathrm{E}} \\ -J_{\mathrm{M}} \end{pmatrix}$$

Complexified gauge coupling:

$$\tau = \frac{\theta}{2\pi} + \mathrm{i}\frac{2\pi}{e^2} \qquad \text{transforms as} \qquad \tau \mapsto \frac{a\tau + b}{c\tau + d}$$

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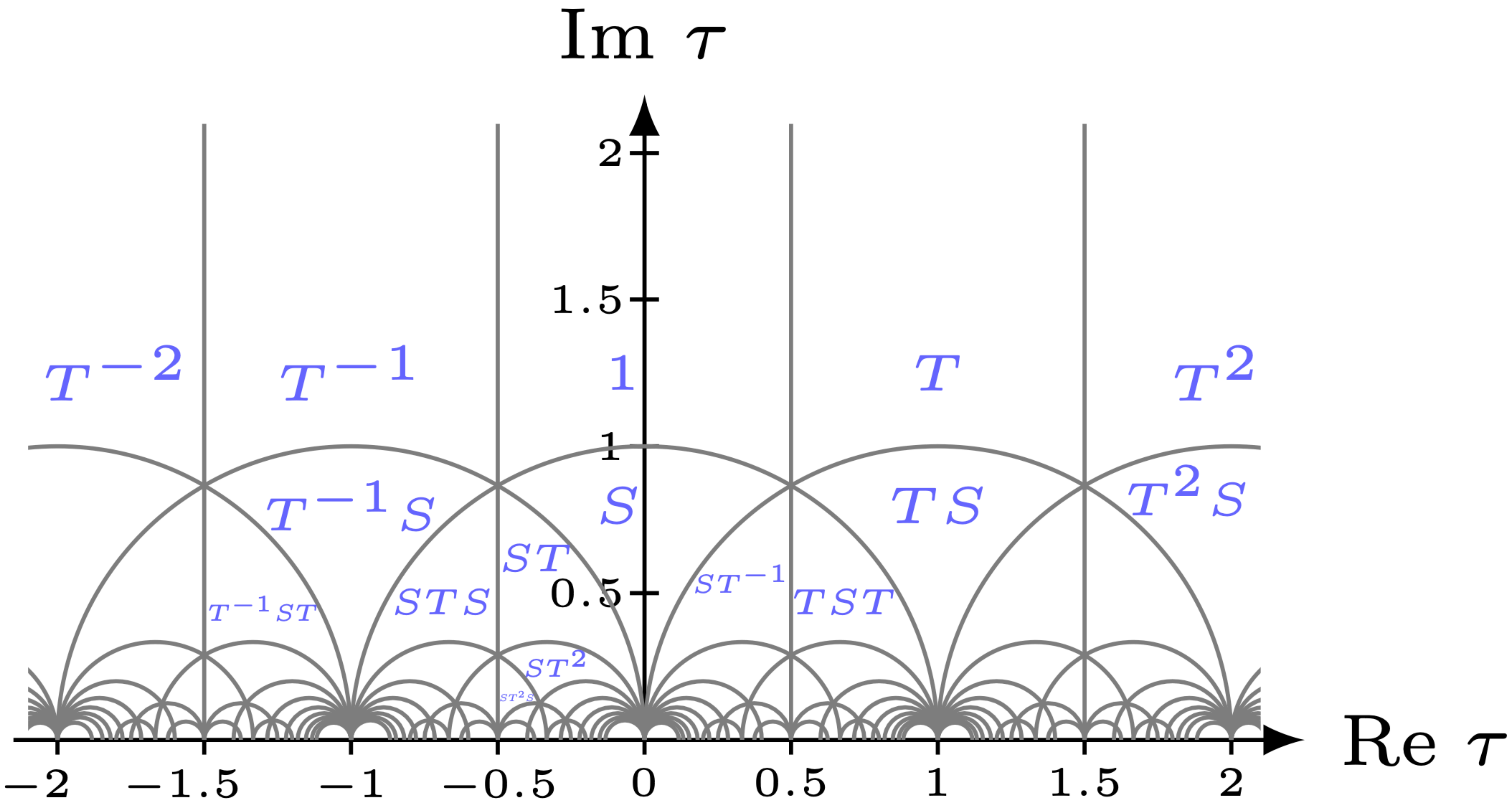


Standard axion couplings

The standard axion coupling has a correspondence between the axion gauge symmetry $\theta(x) \mapsto \theta(x) + 2\pi$ and the $\mathrm{SL}(2,\mathbb{Z})$ transformation

$$\tau \mapsto \tau + n$$

which is the element $T^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ of $\mathrm{SL}(2,\mathbb{Z})$.

This leaves the gauge field A invariant but changes the magnetic gauge field:

$$A_{\mathrm{M}} \mapsto A_{\mathrm{M}} + nA$$

This monodromy is the **Witten effect:** magnetically charged objects acquire *n* units of electric charge when the axion traverses its period.

SL(2,Z)-twisted axion couplings

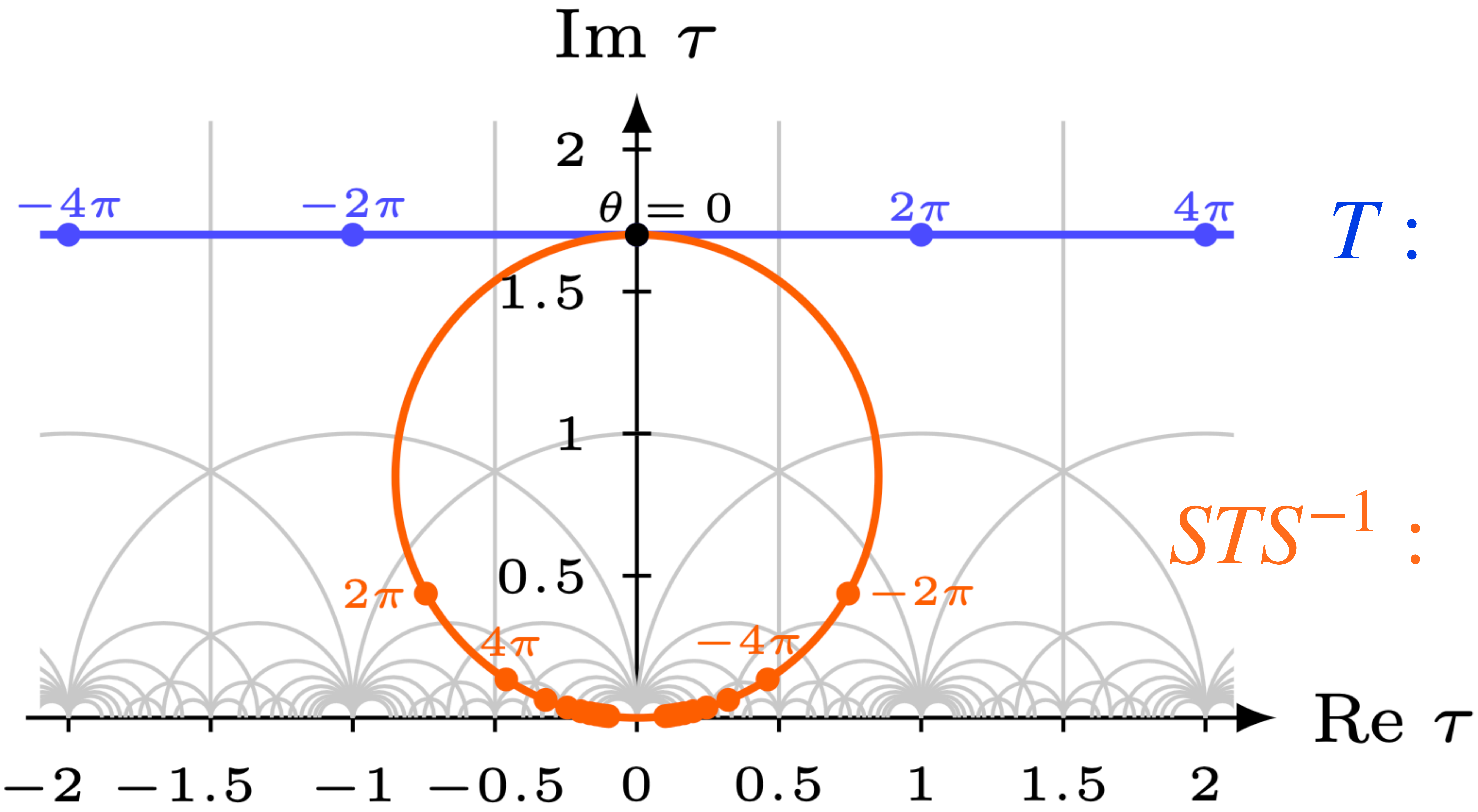
The obvious way to write a theory consistent with the gauge symmetry $\theta(x) \mapsto \theta(x) + 2\pi$ is to have $\tau(\theta + 2\pi) = \tau(\theta)$. But we know that Chern-Simons couplings can have the monodromy $\tau(\theta + 2\pi) = \tau(\theta) + n$.

Why not something even more general?

Combine the axion shift with a general EM duality transformation:

$$\tau(\theta + 2\pi) = \frac{a\tau(\theta) + b}{c\tau(\theta) + d}$$

Example SL(2,Z) monodromies



$$T: \quad \tau \mapsto \tau + 1$$

$$STS^{-1}: \; \tau \mapsto \frac{\tau}{1-\tau}$$

Non-standard axion couplings: example

The monodromy $\tau \mapsto \dfrac{\tau}{1-\tau}$ corresponds to couplings

$$\int \mathrm{d}^4x \left(-\frac{1}{4e^2} \frac{1}{1+\frac{\theta(x)^2}{e^4}} F_{\mu\nu}F^{\mu\nu} \right) + \left(\frac{2\pi}{e^2}\right)^2 \int \theta(x) F \wedge F \frac{1}{1+\frac{\theta(x)^2}{e^4}}$$

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Instead of coupling $\propto e^2$, it's $\propto 1/e^2$ after canonically normalizing.
Fits expectation for "magnetic dual" of standard axion coupling.

Can be shown to match Sokolov, Ringwald 2205.02605 equations of motion after appropriate field redefinitions.

The phenomenological problem

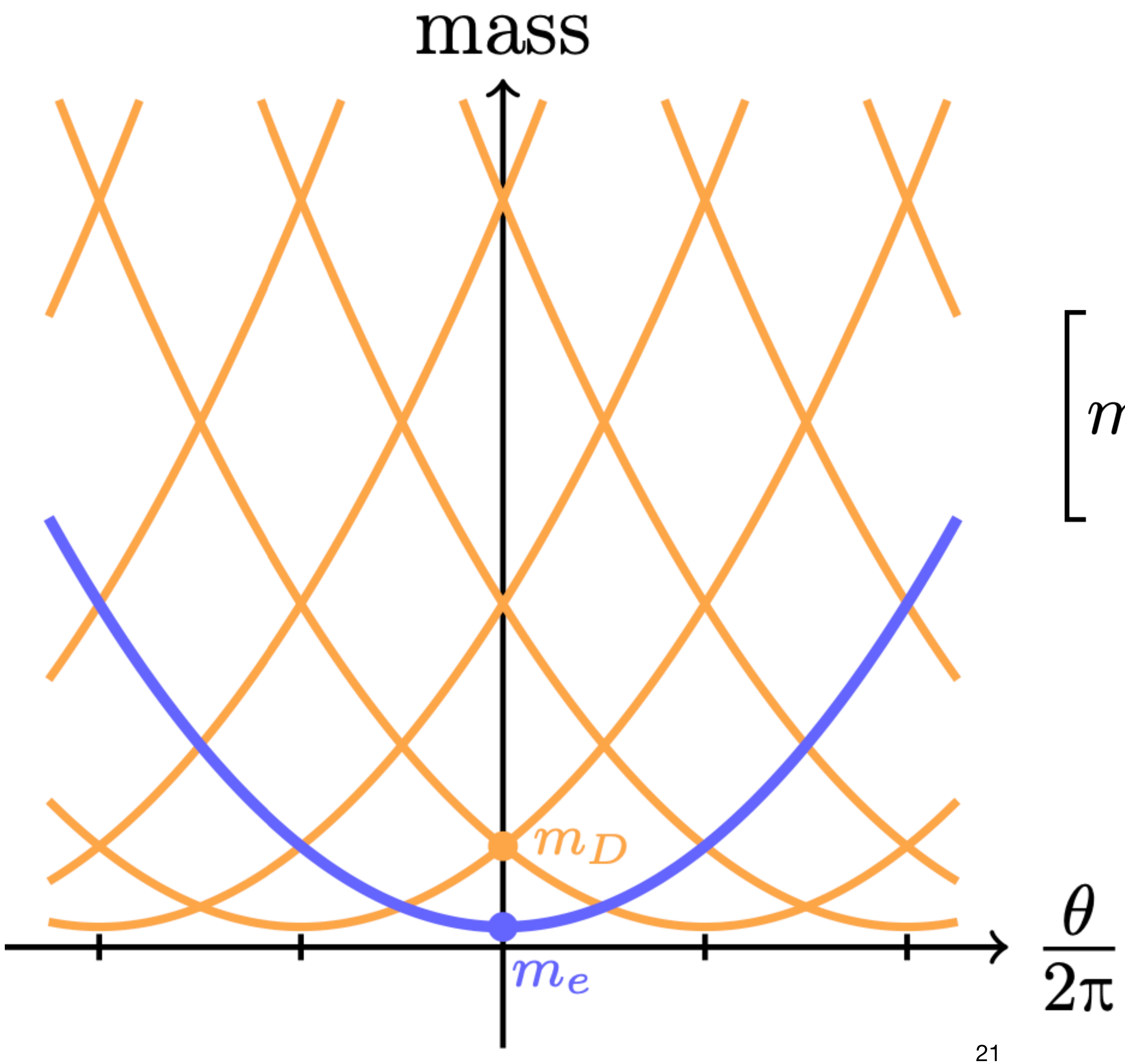
All this is fine in the abstract, coupling an axion to free Maxwell theory. (Actually not quite: $\mathrm{SL}(2,\mathbb{Z})$ can be anomalous. Let's set that aside.)

However, unlike $T^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$, a general $\mathrm{SL}(2,\mathbb{Z})$ transformation will modify not just the magnetic field strength F_M but the standard one F:

$$F \mapsto cF_\mathrm{M} + dF$$

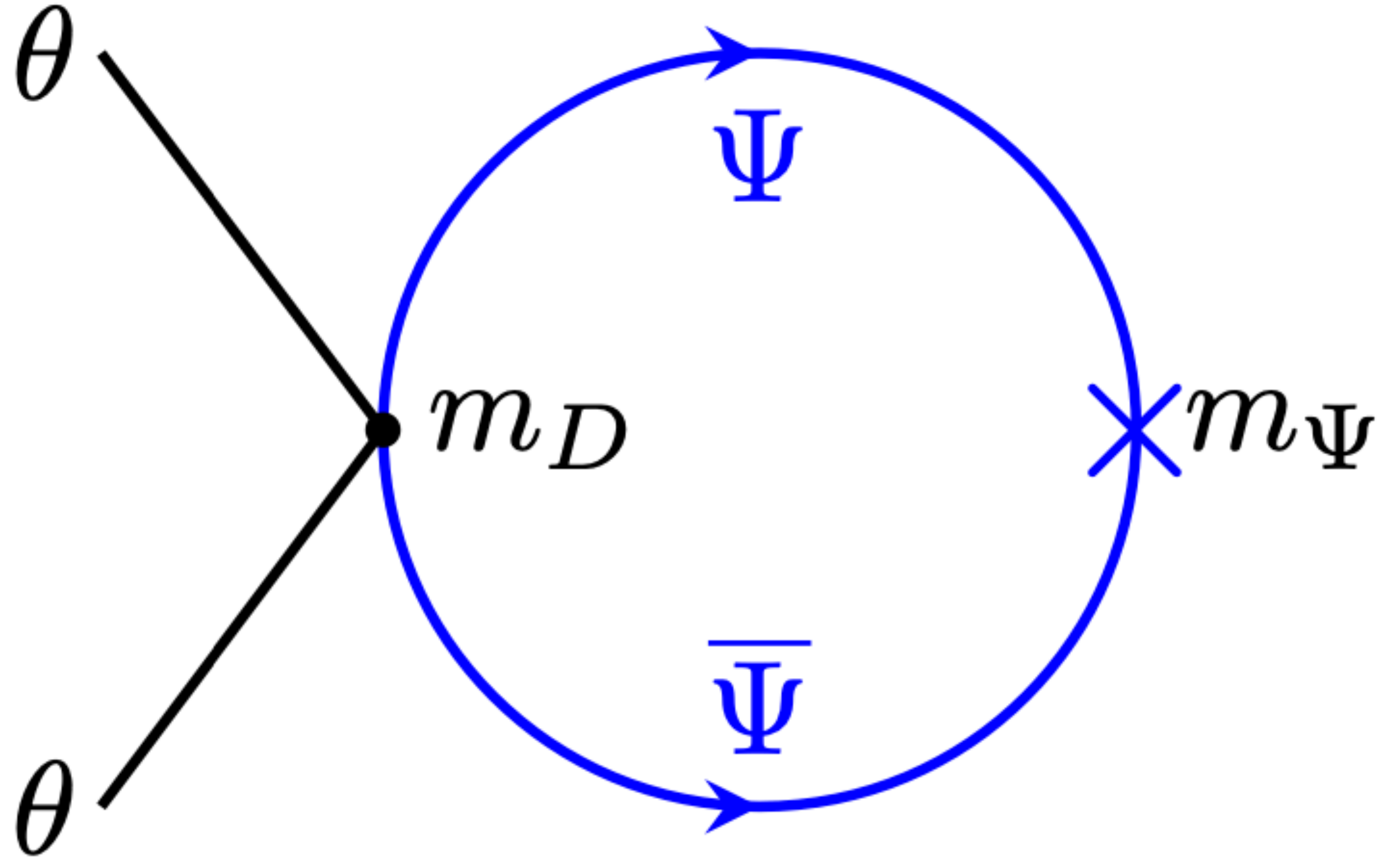
This causes a **dual Witten effect**: the axion monodromy causes every **ordinary electrically charged particle** to become **a dyon with magnetic charge.** The electron, the top quark, etc. — all have dyonic cousins.

Dual Witten effect monodromy



$$\left[m_\Psi + m_D\left(n - \frac{\theta}{2\pi}\right)^2 + \cdots\right]\overline{\Psi}\Psi$$

Axion mass from fermion loop



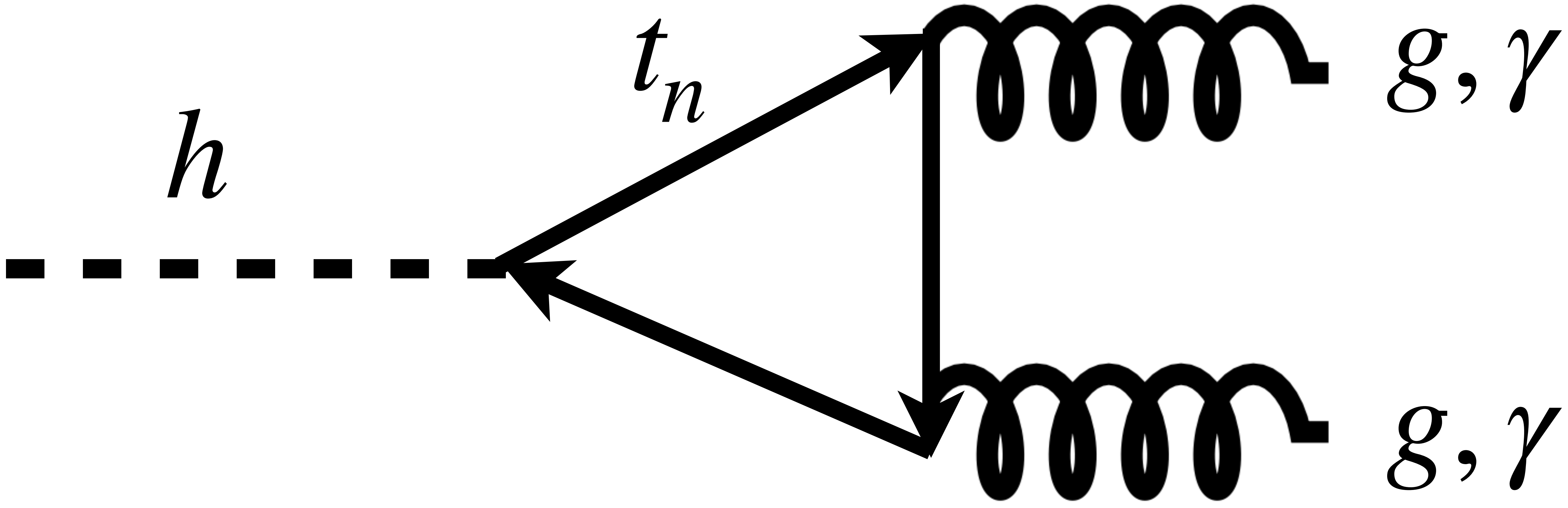
$$m_\theta^2 \sim \frac{1}{16\pi^2} m_\Psi m_D \frac{\Lambda^2}{f^2} \log \frac{\Lambda}{m_\Psi}.$$

from top quark: $m_D \sim \Lambda \sim 1\,\mathrm{TeV}$

$$m_\theta \sim 30\,\mathrm{eV}\,\frac{10^{12}\,\mathrm{GeV}}{f}.$$

Easily overwhelms QCD contribution to potential.

Dyons modify higgs physics



Predict large deviations in the (well-measured) couplings of the Higgs boson to gluons and photons. Simply not viable.

Conclusions

- Experimental signatures of axions depend on the Chern-Simons coupling to photons, which is well-known to be quantized.
- However for the minimal gauge group (with $\mathbb{Z}_6$ quotient), the quantization turns out to depend nontrivially on the gluon coupling, because of field configurations with correlated topological charge.
- Non-standard couplings with $\mathrm{SL}(2,\mathbb{Z})$ monodromies are possible in the abstract, but not for the Standard Model: the dual Witten effect rules them out.
- Sikivie wrote down the correct equations in 1984. These, not non-standard alternatives, should guide experiments.